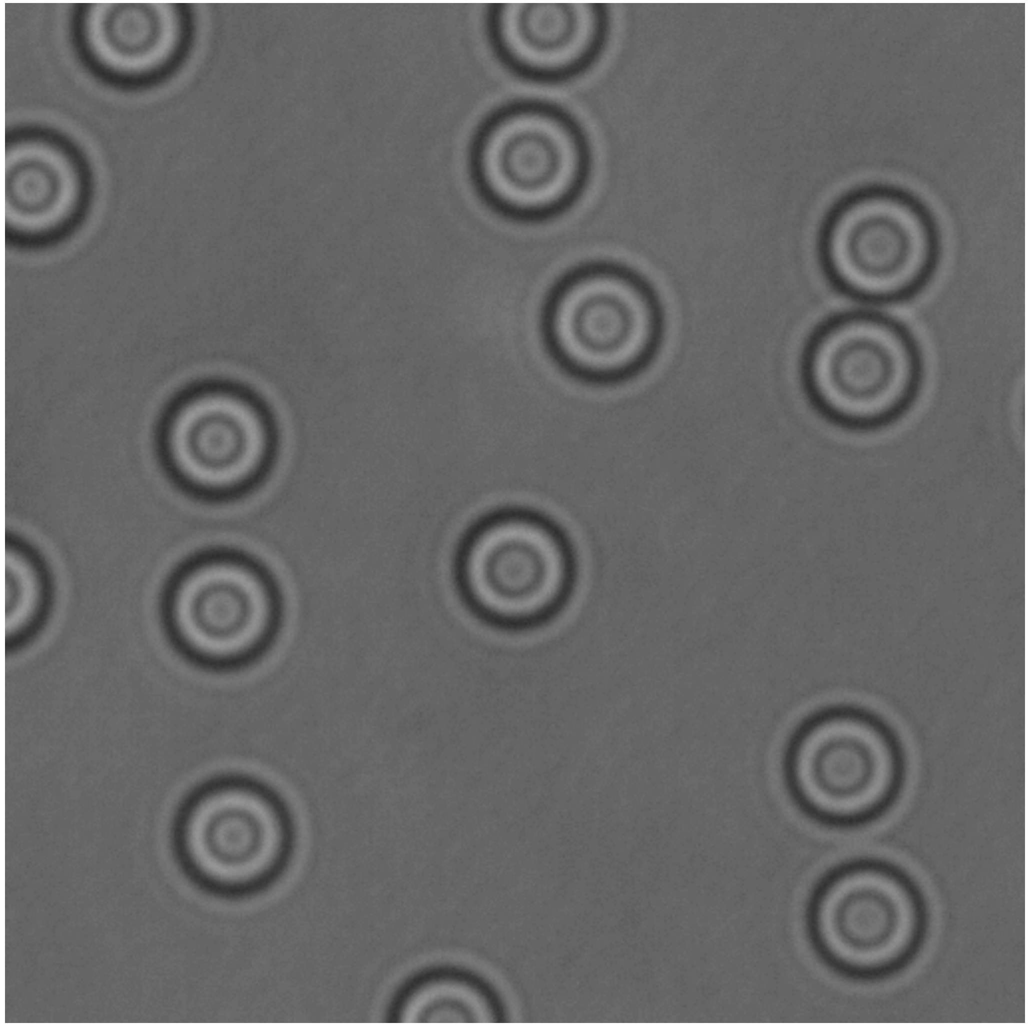
Dynamics of perfect gases : from atoms to fluid models

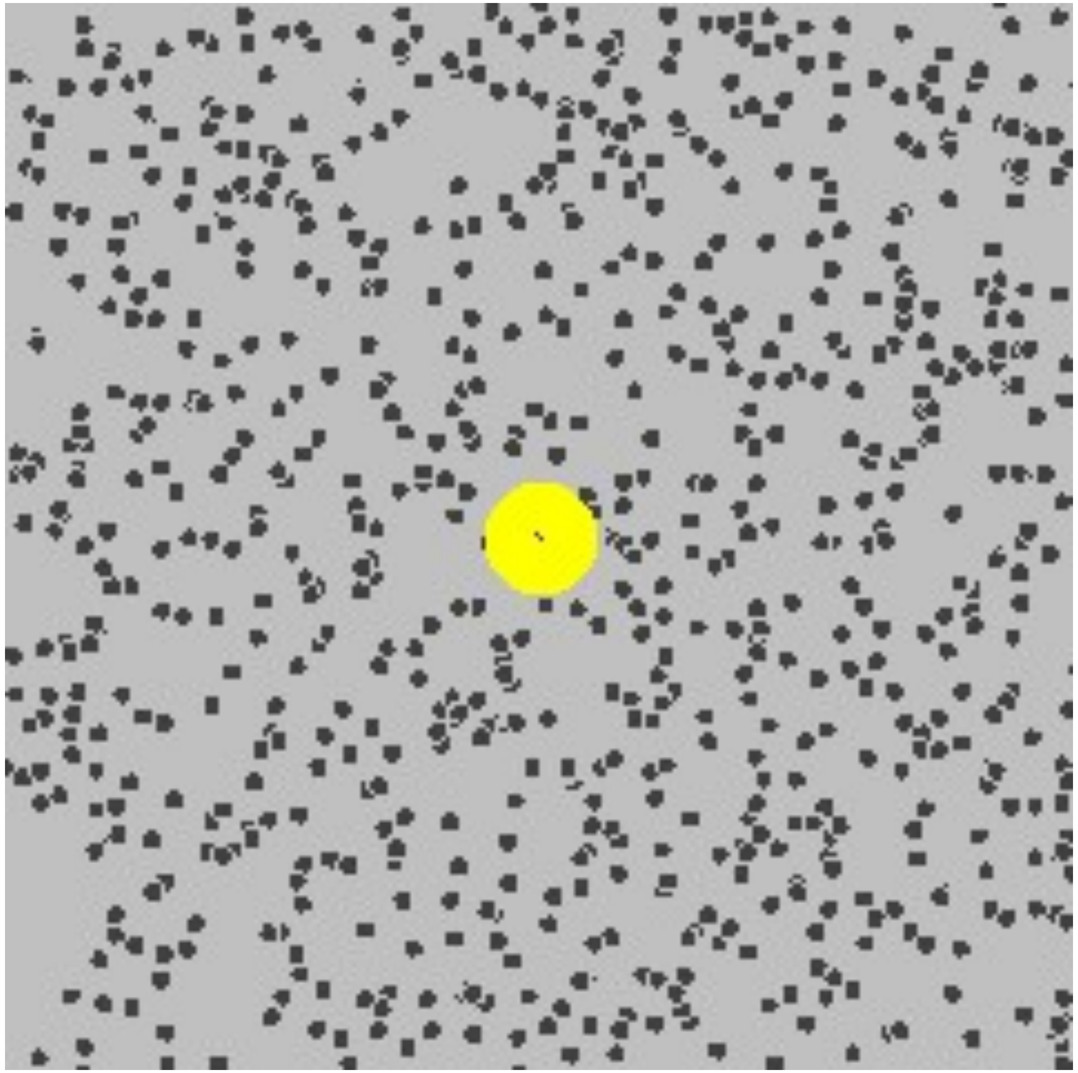
A problem more than a century old!

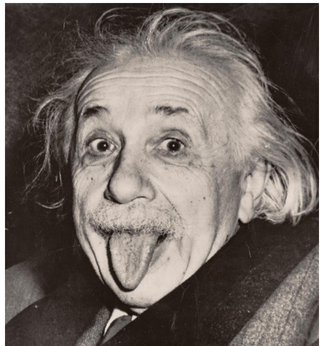
1. A simple case :

the motion of a pollen grain

« A brief account of microscopical observations on the particles contained in the pollen of plants ; and on the general existence of active molecules in organic and inorganic bodies » (R. Brown)







a small particle suspended in a fluid should be agitated by collisions with molecules

Microscopic model
(Einstein, Perrin)

- Collisions with microparticles
- Random distribution of microparticles
- No feedback

Randomness?

Size of microparticles negligible

Brownian motion
(Wiener, Levy)

- Continuous trajectories
- Independent increments
- Gaussian increments

$$\mathbb{E}[(x(t) - x(s))^2] = \sigma |t - s|$$

Long time behaviour of observables

Macroscopic model
(Fourier, Fick)

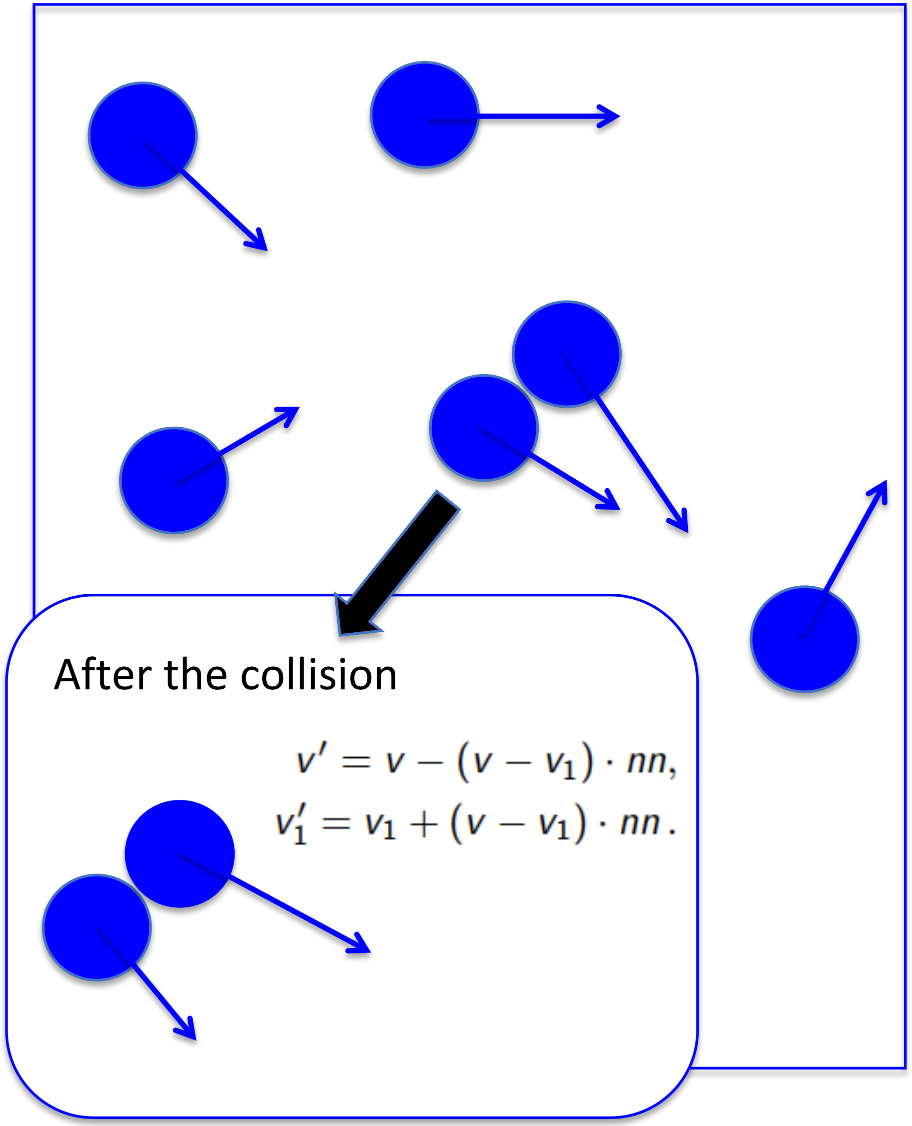
- Density, temperature
- Diffusion equation

$$\partial_t \Theta - \kappa \Delta_x \Theta = 0$$

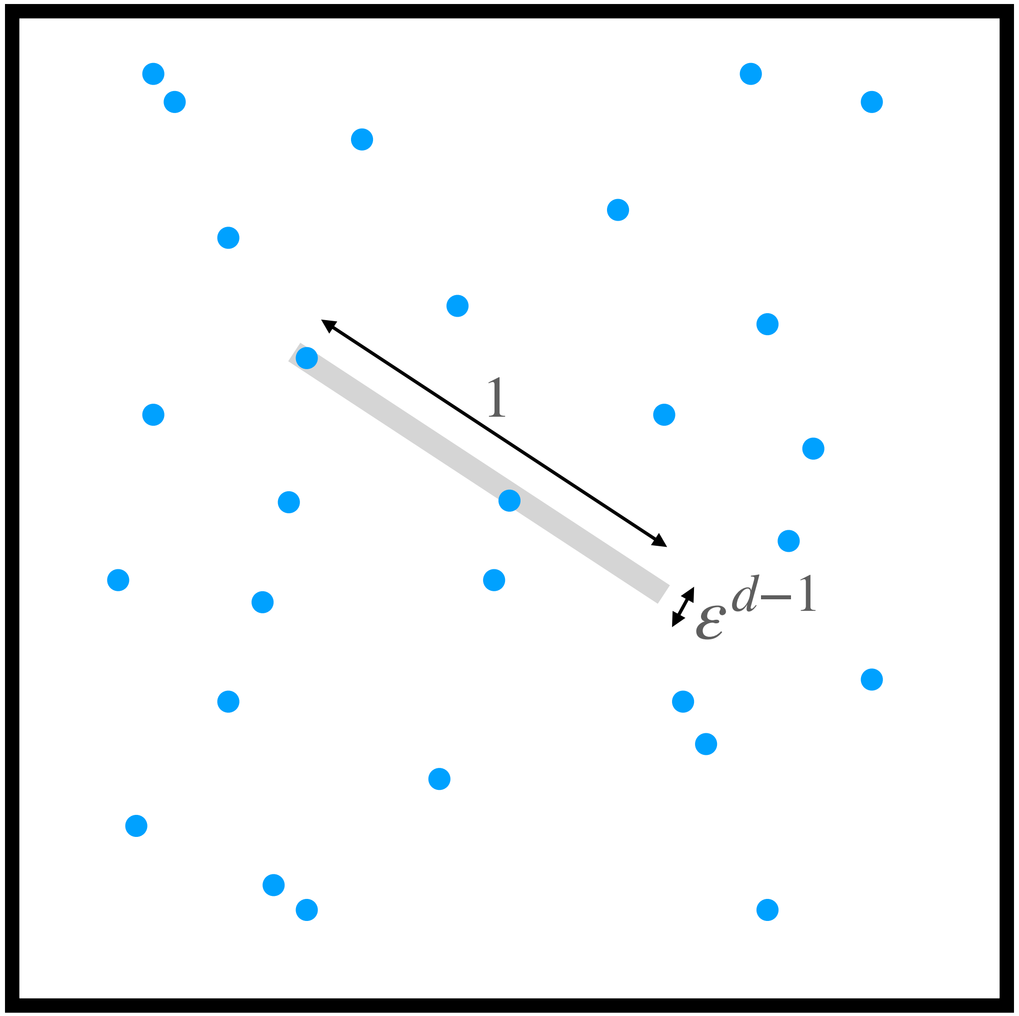
2. Perfect gases :

From Boltzmann to Lanford

A gas is a collection of interacting atoms.
To simplify, we consider contact interactions.



The particularity of perfect gases is that their atoms are very weakly bound. In dilute regime, the mean free path $(\mu\varepsilon^{d-1})^{-1}$ is of order 1.



<u>Microscopic model</u>

- Elastic collisions
- Deterministic, reversible dynamics
- Chaotic initial data

<u>Macroscopic model</u>

(Navier-Stokes-Fourier)

- Density, bulk velocity, temperature
- Irreversible

Low density limit (Lanford)

<u>Boltzmann equation</u>

- Statistical model
- Nonlinear pointwise interactions
- Chaotic distribution

$$\partial_t f + v \cdot \nabla_x f = Q(f,f)$$

$$Q(f,f) = \int \Big(f(v')f(v'_*) - f(v)f(v_*) \Big) d\mu_v(v_*, \omega)$$

Fast relaxation limit (Hilbert)

Chaos is propagated for short kinetic times!

3. Recent developments

With T. Bodineau, I. Gallagher, S. Simonella

A global statistical picture

The convergence to the Boltzmann equation when $\mu >> 1$ has to be understood as a law of large numbers, describing the almost sure dynamics.
 Propagation of chaos is satisfied at leading order, but correlations (which can be studied by cumulant techniques) induce fluctuations.

Central limit theorem : typical fluctuations of the empirical measure are of order $O(\mu^{-1/2})$ and are governed asymptotically by a stochastic Boltzmann equation.
 Dynamical noise appears spontaneously, it comes from the sensitivity of the dynamics to microscopic details of the initial data..

Large deviation principle : the probability to observe atypical dynamics is exponentially small. The large deviation functional satisfies some Hamilton-Jacobi equation.
 Stochastic reversibility is retrieved at this level.

Long time behavior and hydrodynamic limits

The fluctuation field of the hard sphere system at equilibrium converges in law for all kinetic times and even slowly diverging times to the Gaussian process, solution of the fluctuating Boltzmann equation.

In the fast relaxation limit, with a parabolic rescaling of space and time, the incompressible hydrodynamic fields converge in law to Gaussian processes, solutions to the fluctuating Stokes-Fourier equations.